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SM3 13.1 NH: Arithmetic Sequences

A sequence is an ordered list. Sequences are useful for studying patterns in mathematics. Sequences are usually named with a single lower-case letter and are delimited by commas. Sequences can have any number of terms, which means they can be either finite or infinite.

We refer to individual terms of a sequence by using the name of the sequence with a subscripted index. So, a_4 represents the 4th term of a sequence called a.

Example: $p = 7, \pi, 100, 0.25, 11$
p is a finite sequence because it has 5 terms.

$p_1 = 7, p_2 = \pi, p_3 = 100, p_4 = 0.25, p_5 = 11$

Example: $q = 2, 5, 8, \ldots$
q is an infinite sequence because it has infinitely many terms.

$q_1 = 2, q_2 = 5, q_3 = 8, \ldots$

Mathematicians use ellipses (…) to mean "and so forth". Ellipses can be used at any location in a sequence to continue the sequence. When present at the edge of a sequence, ellipses indicate that the sequence is infinite in the direction of the ellipses.

A sequence is considered **arithmetic** when a common difference exists from term to term. We reserve lower-case d to mean the common difference of a sequence. You can add d to get from any term to the next term or subtract d to get from any term to the previous term. We call this fact the recursive rule, which states that if you want to find any term in an arithmetic sequence, you can just add the common difference to the previous term.

Recursive Rule: $a_n = a_{n-1} + d$

Example: $s = 157,161,165,169, \ldots$ Use the recursive rule to find s_7.

We can only use the recursive rule if the sequence is arithmetic, which means it has a common difference. The common difference of sequence s is ____.

Use the recursive rule by adding the common difference to any term to find the next term. Keep going until you get the value you're looking for.

$s_4 = 169$	Given
$s_5 = \quad + \quad =$	Use the recursive rule with $n = 5$
$s_6 = \quad + \quad =$	Use the recursive rule with $n = 6$
$s_7 = \quad + \quad =$	Use the recursive rule with $n = 7$

Students typically do not find using the recursive rule to be challenging because the recursive rule feels like common sense. Unfortunately, it can be tedious. Imagine if we wanted to find $s_{123823567}$.

Often, it is too much work to find a term recursively. Let's discover a general solution to finding the nth term of a sequence.

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Find $s_{123823567}$ of $s = 157, 161, 165, 169, \ldots$

Sequence s is built by adding 4 to get to the next term.

To get to the second term, we added 4 to the first term.

To get to the third term, we added $4 + 4$ (that is 2 copies of 4) to the first term.

To get to the fourth term, we added $4 + 4 + 4$ (that is 3 copies of 4) to the first term.

⋮

To get to the 123823567th term, we will add   copies of 4 to the first term.

⋮

In general, to get to the nth term, we will add   copies of 4 to the first term.

General Rule: $a_n = a_1 + (n - 1)d$

$s_{123823567} = 157 + (\qquad\qquad)4 =$

While the numbers involved with the calculation were large, at least we it was just one calculation!

Should you use the recursive rule or general for finding terms in a sequence?

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Homework Problems

Let $v = \frac{1}{2}, 9, 32, \ln 17, 4, e^3$. Evaluate the following:

1) Is v arithmetic; if so, find the common difference, d.

2) $v_1 =$

3) $v_3 =$

4) $v_4 =$

Let $a = 2, 7, 12, 17, 22, 27, 32, 37$. Evaluate the following:

5) Is a arithmetic; if so, find the common difference, d.

6) $a_1 =$

7) $a_3 =$

8) $a_5 =$

9) $a_9 =$

Let $b = 20, 13, 6, -1, -8, \dots$. Evaluate the following:

10) Is b arithmetic; if so, find the common difference, d.

11) $b_1 =$

12) $b_6 =$

13) $b_{10} =$

14) $b_{415} =$

Let $c = 0.3, 1.1, 1.9, \dots, 80.3, \dots$. Evaluate the following:

15) Is c arithmetic; if so, find the common difference, d.

16) $c_1 =$

17) $c_5 =$

18) $c_n = 80.3; n =$

19) $c_{29} =$

Let $q = \dots, 2000, 2004, 2008, \dots$. Evaluate the following:

15) Is q arithmetic; if so, find the common difference, d.

16) $q_1 =$

17) Describe a real world context for what q could represent.

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William acquires a bag of candy on the evening of Halloween. He eats his candy during the month of November in the following manner: he eats 3 pieces of candy for each day of the month it is (i.e., on the 1st day of November, he eats 3 pieces of candy. On the 2nd day of November, he eats 6 pieces of candy, etc.). Let sequence c represent how many candies are eaten where c_n represents the number of candies eaten on the n^{th} day of November.

18) Is c arithmetic; justify your response.

19) How many pieces of candy does William eat on November 17th?

20) How many pieces of candy does William eat on November 25th?

Mr. Wytiaz decides to lose weight and signs up for a membership at *Linear Weightloss Gym*. After consulting with a personal trainer, Mr. Wytiaz's goal is to lose at least 40 of his 200 pounds. By performing an aerobic workout followed by a kung fu sparring match with Ms. Shaw, he is able to lose $\frac{3}{4}$ of a pound each day. Let sequence w represent the weights of Mr. Wytiaz where w_n represents his weight after his n^{th} session.

21) Is w arithmetic; justify your response.

22) $w_1 =$

23) $w_6 =$

24) How many sessions until Mr. Wytiaz loses 40 lbs?

The sum of the interior angle measurements of a triangle is 180°, of a convex quadrilateral is 360° and of a convex pentagon is 540°. Let sequence a_n represent the sum of interior angle measurements of a convex polygon with n sides.

25) $a_3 =$

26) $a_4 =$

27) $a_5 =$

28) $a_1 =$

29) What does a_1 represent?

30) Find the sum of the interior angles of a convex dodecagon ($12gon$).

31) As a_3 represents the polygon with the fewest sides possible, how could we alter our choices of a and n to not include meaningless representations?

32) What is the strongest argument for making the above alteration? What is the strongest argument against making the above alteration?